The Starring Role of Quantifiers
in the History of Formal Semantics
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Abstract
The history of formal semantics is a history of evolving ideas about logical form, linguistic form, and the nature of semantics. This paper emphasizes parts of the history of semantics where quantifiers played a major role, including the “Linguistic Wars” of the late 1960’s and the conflicts in the philosophy of language between the Ordinary Language philosophers and the Formal Language philosophers. Both conflicts resulted in part from the mismatch between first-order logic and natural language syntax. Both were resolved in part once Montague applied his higher-order typed intensional logic to the analysis of natural language, as illustrated most vividly by the treatment of noun phrases as generalized quantifiers. And quantifiers have played a central role in a number of key subsequent developments in the formal semantics of natural language.

Keywords: formal semantics, formal pragmatics, logical form, linguistic form, quantifiers, Montague, Montague grammar, compositionality, history of semantics

1. Introduction

There have been centuries of study of logic and of language. Many philosophers and logicians have argued that natural language is logically deficient, or even that “natural language has no logic”. And before the birth of formal semantics in the late 1960’s, both linguists and philosophers were mostly agreed, for very different reasons, that what logicians meant by “semantics” had no relevance for the study of natural language. But logicians and philosophers of language, even those who regarded natural languages as “illogical” in various ways, made crucial advances in semantic analysis that paved the way for contemporary formal semantics.

The logician and philosopher Richard Montague argued that natural languages do have a very systematic semantic structure, but that it can be understood only if one uses a rich enough logic to mirror the rich type structure that he saw in natural languages. So changing views of the relation between language and logic have often involved changing views of logic itself, and of linguistic structure.

In this paper¹ I’ll review some of this background and sketch developments in the growth of formal semantics and formal pragmatics, focusing on crucial turning points in the history of semantics where quantifiers have played a major role. One example: the theory Chomsky described in his 1965 Aspects of the Theory of Syntax, where meaning was

¹ This paper overlaps with and builds on (Partee, 2011), which provides background for the issues discussed here. Both are part of my work for a book in progress on the history of formal semantics. For the early history of quantification, I have made great use of (Westerståhl, 2011) and (Peters & Westerståhl, 2006), and other sources mentioned in the text. I am grateful for comments and discussion to Paul Pietroski, Alexander Williams, Joshua Stuart Falk, Jon Michael Dunn, participants in Angelika Kratzer’s semantics proseminar on quantification in Fall 2011, the participants in the conference “Logica 2012” in Hejnice, Czech Republic, in June 2012, and the audience at my second Baggett Lecture at the University of Maryland in November 2012. All mistakes are my own.
determined at Deep Structure and transformations were meaning-preserving, ushered in a brief “Garden of Eden” period; what led to expulsion from the Garden and to the Linguistic Wars was (oversimplifying only a bit) linguists’ “discovery” of quantifiers. I’ll describe this and a number of other crucial points, some earlier and some later. The history of formal semantics is much more than the history of treatments of quantifiers, but their story is an important and fascinating chapter.

2. Semantics in linguistics and the “discovery” of quantifiers

Semantics tended to be rather neglected in early and mid-20th century American linguistics. There were several reasons. There had been rather little semantics in early American anthropological linguistics, since in doing linguistic fieldwork one had to start with phonetics, then phonology, then morphology, occasionally a little syntax, and rarely any semantics beyond making dictionaries, or in working out particular lexical domains such as kinship terms. And the behaviorists viewed meaning as an unobservable aspect of language, not fit for scientific study, which influenced the structuralists. And Quine had strong skepticism about the concept of meaning, and had some influence on Chomsky.

At the same time there was great progress in semantics in logic and philosophy of language, but that was relatively unknown to most linguists.

In 1954, Yehoshua Bar-Hillel wrote an article in *Language* (Bar-Hillel, 1954) inviting cooperation between linguists and logicians, arguing that advances in both fields made the time ripe for combining forces to work on syntax and semantics together. But Chomsky (1955) rebuffed the invitation, arguing that the artificial languages invented by logicians were too unlike natural languages for the methods the logicians had developed to have any chance of being useful for developing linguistic theory.

In *Syntactic Structures* (Chomsky, 1957), Chomsky is quite ambivalent about semantics. He argues that semantic notions are of no use in constructing a grammar, emphasizing that intuitions of grammaticalness are distinct from intuitions of meaningfulness. But at the same time he holds that one test of a good syntax is that it should provide a good basis for a good semantics (if we had any idea how to study semantics). And he argues that transformational grammar is a positive step in that direction, since it uncovers differences at the “transformational level” (what would later be reworked as “deep structure”) that are obscured in the output (later “surface structure”).

But Chomsky also notes that transformations sometimes change meaning. “… we can describe circumstances in which a ‘quantificational’ sentence such as [(1a)] may be true, while the corresponding passive [(1b)] is false, under the normal interpretation of these sentences – e.g., if one person in the room knows only French and German, and another only Spanish and Italian. This indicates that not even the weakest semantic relation (factual equivalence) holds in general between active and passive.” (pp. 100-101)

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2 I focus here on linguistics in America, since America is where Montague grammar and formal semantics began. In my book in progress, I will say a bit about the rather different scene in European linguistics.

3 When David Kaplan heard about Chomsky’s reply to Bar-Hillel, he said it reminded him of Quine’s vehement rejection of Kripke’s work on modal logic (David Kaplan, p.c. January 2011).
(1)  a. Everyone in this room knows at least two languages.
    b. At least two languages are known by everyone in this room

In later years, those judgments about (1) came to be questioned; some argued that (1b) is ambiguous, some argued that both are. Chomsky himself noted problems with the judgments and their diagnosis in (Chomsky, 1965). Difficulties with such data continued for many years. Over time, linguists have developed more subtle ways to get data than just asking about their own or their consultants’ intuitions. But the unclear relation between English (and not only English) syntax and quantifier scope has continued to be a topic of much study, many proposals, and little consensus.

**Early semantics in generative grammar.** Katz and Fodor (1963) added a semantic component to Chomsky’s generative grammar. They addressed the “Projection Problem”, i.e. compositionality: how to get the meaning of a sentence from meanings of its parts. At that time, “Negation” and “Question Formation” were transformations of declaratives: they were prime examples of meaning-changing transformations. So at that time, it was accepted that meaning depended on the entire transformational history.

In a theoretically important move, related to the problem of compositionality, Katz and Postal (1964) made the innovation of putting such morphemes as Neg(ation) into the Deep Structure, as in (2), arguing that there was independent syntactic motivation for doing so; they hypothesized that transformations never change meaning, and that meaning could be determined on the basis of Deep Structure alone. The revised Negation transformation T-NEG did not change meaning; its job was to put the negative morpheme into its appropriate form and position in the Surface Structure, the level that provides the input to phonological rules that determine how the sentence is pronounced.

(2) \[ \text{Neg} \ [\text{Mary} \ [\text{has} \ [\text{visited Moscow}]]] \Rightarrow \text{T-NEG} \\
    \ [\text{Mary} \ [\text{has not} \ [\text{visited Moscow}]]] \]

In *Aspects of the Theory of Syntax* (Chomsky, 1965), Chomsky tentatively accepted Katz and Postal’s hypothesis. The architecture of the theory -- syntax in the middle, mediating between semantics on one side and phonology on the other -- was elegant and attractive. This big change in architecture rested on the claim that transformations should be meaning-preserving. This led to what I call the “Garden of Eden” period around 1965, when there was widespread optimism about the Katz-Postal hypothesis, and the syntax-semantics interface was believed to be relatively straightforward (even without having any really good ideas about the nature of semantics.)

What happened to upset that lovely view? Although of course there were multiple factors, I think it’s fair to focus on one salient issue: linguists discovered quantifiers (Bach, 1968; Karttunen, 1968, 1969; Lakoff, 1968; McCawley, 1971). Transformations that preserved meaning (more or less) when applied to names clearly did not when applied to some quantifiers. Clear examples come from “Equi-NP Deletion” (Rosenbaum, 1967), the transformation that applied to (the structure underlying) (3a) to give (3b).

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4 Quantifier scope ambiguity in natural language remains a big and important topic; I will neglect it here for reasons of space, but in the planned book I will discuss at least 10 proposals for analyzing it.
When the identical NPs are names, the transformation preserves meaning. But applied to sentences with quantifiers, it has the unwanted result of deriving (4b) from (4a).

(4) a. Everyone wants everyone to win.
   b. Everyone wants to win.

It was a surprising historical accident that the behavior of quantifiers was not really noticed until the Katz-Postal hypothesis had for most linguists reached the status of a necessary condition on writing rules. Here are a few more examples of derivations that would have been given in Chomsky’s 1965 “standard” theory; I doubt that the Katz-Postal hypothesis would have been suggested if these had been noticed earlier.

(5) a. Every man voted for himself. FROM:
   b. Every man voted for every man.
(6) a. All pacifists who fight are inconsistent. FROM:
   b. All pacifists fight. All pacifists are inconsistent.
(7) a. No number is both even and odd. FROM:
   b. No number is even and no number is odd.

The problems illustrated by examples (4-7) showed the untenability of the Katz-Postal hypothesis combined with the Aspects theory of syntax, and led to expulsion from the Garden of Eden and to the “Linguistic Wars” (Harris, 1993; F. Newmeyer, 1980) between the Generative Semanticists and the Interpretive Semanticists. Generative Semanticists held onto the goal of compositionality and pushed the ‘deep’ structure deeper, making it a kind of “logical form”. Chomsky had been tentative about adopting the Katz-Postal hypothesis in the first place, and valuing syntactic autonomy more highly, abandoned it. The linguistic wars raged from the late 1960’s into the mid 1970’s.

What were the early linguistic notions of ‘logical form”? Generative Semanticists (Lakoff, Ross, McCawley, Postal, and others) (see F. J. Newmeyer, 1996), argued that in order for deep structure to capture semantics, it needed to be deeper, more abstract, more like “logical form”, which for linguists meant first-order-logic. In fact, both linguists and philosophers who worried about the semantics of quantified sentences before Montague’s work thought of “logical form” in terms of first-order logic. But given that generalized quantifiers were only developed starting in the late 1950’s, that could hardly have been otherwise. I return in Section 4 to the role of generalized quantifiers in reconciling the apparent mismatches between “linguistic form” and “logical form” of sentences with quantifiers. But first we need a quick review of some of the crucial developments from Aristotle to Frege and Tarski.

3. Developments in Logic
This section is short, since most of this material is familiar to the *Logica* readership. I will just review some relevant history of quantifiers that forms part of the background for the development of formal semantics.

When Aristotle (384–322 B.C.E.) invented logic, he focused on quantification; operators like *and* and *or* were added by the Stoics. Implicit in Aristotle’s syllogistic is a semantics for the quantifiers. Each of the four quantifier expressions can be seen as standing for a binary relation between properties:

\[(8) \begin{align*}
(i) \text{ all } (A, B) & \iff A \subseteq B \\
(ii) \text{ some } (A, B) & \iff A \cap B \neq \emptyset 
\end{align*}\]

In hindsight, this is close to the idea of Generalized Quantifiers. But the idea of giving a semantic value to the quantifiers themselves was not explicitly developed until much later, really not until Frege. In the Middle Ages there was a lot of work trying to figure out meanings for expressions like *all men* or *some man*, sometimes syncategorematically (Albert of Saxony analyzed *all* and *some* via conjunctions and disjunctions), sometimes categorematically (giving them independent meanings) with convoluted theories.

Leibniz may have been the first to use bound variables, but it was in his integral calculus, not in logic. Those variables were intrinsically bound, not replaceable by constants. A different use of variables was already in use in algebra, in formulas like (9).

\[(9) x + (y + z) = (x + y) + z \quad \text{Law of Associativity}\]

These variables could be substituted for by constants; they were implicitly bound by universal quantifiers.

But those two uses were not united and generalized until Frege.

The word 'quantifier' appears first in De Morgan (1862), as an abbreviation for Hamilton’s 'quantifying phrase'. The American philosopher and logician C.S. Peirce (1839-1914) is often credited with developing the theory of relations (Burris, 2009).

**Frege.** The greatest foundational figure for formal semantics is Gottlob Frege (1848-1925). His crucial ideas include the Compositionality Principle and the idea that *function-argument* structure is the key to semantic compositionality.

**The Principle of Compositionality:** The meaning of a complex expression is a function of the meanings of its parts and of the way they are syntactically combined.

One of Frege’s great contributions was the logical structure of quantified sentences. That was part of the design of a “concept-script” (*Begriffsschrift*), a “logically perfect

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5 For help with this section I am indebted to more conversations and sources than I can remember; my sources include at least (Cocchiarella, 1997; Stanley, 2008) and conversations with Dagfinn Føllesdal, Joseph Almog, David Kaplan, and others, in addition to my own rather eclectic education in philosophy.

language” to satisfy Leibniz’s goals; he did not see himself as offering an analysis of natural language, but a tool to augment it, as the microscope augments the eye.

**Does ordinary language ‘have no logic’?** As Russell, Carnap, and Tarski were making advances in logic and the philosophy of language, a war began within philosophy of language, the “Ordinary Language” vs “Formal Language” war. Ordinary Language Philosophers rejected the formal approach, and urged more attention to ordinary language and its uses. Strawson said in ‘On referring’ (Strawson, 1950): “The actual unique reference made, if any, is a matter of the particular use in the particular context; …Neither Aristotelian nor Russellian rules give the exact logic of any expression of ordinary language; for ordinary language has no exact logic.” Russell (1957) replied, “I may say, to begin with, that I am totally unable to see any validity whatever in any of Mr. Strawson’s arguments.” But near the end of the paper, he adds, “I agree, however, with Mr. Strawson’s statement that ordinary language has no logic.”

Russell was not the first logician to complain about the illogicality of natural language. One of his favorite complaints was that English puts phrases like “every man”, “a horse”, “the king” into the same syntactic category as proper names. He considered the formulas of his first-order logic a much truer picture of ‘logical form’ than English sentences.

An exercise I often give my students to help them appreciate Montague’s use of a higher-order typed logic, including generalized quantifiers, is to consider the question of where in Russell’s formula (10), symbolizing *Every man walks*, is the meaning of *every man*?

\[
\forall x \ (\text{man}(x) \rightarrow \text{walk}(x))
\]

The answer is that it is distributed over the whole formula – in fact everything except the predicate *walk* in the formula can be traced back to *every man*. One way to answer Russell is to devise a logic in which the translation of *every man* is a constituent in the logical language. Terry Parsons did it with a variable-free combinatory logic (Parsons, 1968, 1972). Montague did it with a higher-order typed intensional logic (Montague, 1973). Both were reportedly\(^7\) influenced by seeing how to devise algorithms for mapping from (parts of) English onto formulas of first-order logic, thereby realizing that English itself was not so logically unruly. (See also D. Lewis, 1970.) First-order logic has many virtues, but similarity to natural language syntax is not one of them.

**More on Frege and Tarski.** Frege worked out the semantics of free and bound variables, and developed the syntax and semantics of quantifiers as variable-binding operators. And in a sense he did it more compositionally than Tarski. In Tarski’s semantics for quantified sentences, standard in logic textbooks, the quantifier symbols $\forall$ and $\exists$ are not themselves given a semantic interpretation. They are treated syncategorematically: we are given semantic interpretation rules for formulas containing quantifiers. Tarski’s semantics is thus not strictly compositional.\(^8\) Tarski does not get the interpretation of (10) by

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\(^7\) Terry Parsons, p.c. In the case of Montague, a number of people have pointed to the attention to translation between English and logic in (Kalish & Montague, 1964).

\(^8\) This has been reported in various works; I learned it, with some embarrassment, from Tarski (p.c.) after having said, in a talk about Montague Grammar for the Berkeley logic group in the 1970’s where he was
combining the interpretation of the quantifier with the interpretation of the rest of the formula; instead he has a schema that gives the interpretation of (10) by considering satisfaction of the open formula by all possibly values of the variable $x$.

Frege treated the quantifier symbols as categorematic, standing for certain second-order objects (Peters & Westerståhl, 2006, pp35-40). Although his notation was quite different from modern notation, he treated the universal quantifier as a unary second-level operator that applies to a first-level predicate to give a truth-value. Peters and Westerståhl observe that Frege thus invented a kind of generalized quantifier, though it was forgotten until reinvented in a model-theoretic context.

Frege’s universal quantifier was “everything” rather than “every”. We can represent it (not in Frege’s own notation) as a set of sets as in (11) (where $D$ is the universe):

\[\lambda P. \forall x P(x) \quad \text{or} \quad \{P: D \subseteq P\}\]

A sentence like *Every boy walks* would be paraphrased into something like “Everything is such that if it is a boy, it walks.” Thus Frege’s analysis of universal quantifiers had some things in common with later generalized quantifiers, but like Russell, he did not directly analyze NP constituents like *every boy*.

Tarski (1902-1983) developed model theory based in set theory and with it made major advances in providing a semantics for logical languages. Frege had had an absolute notion of truth, and a single domain of all objects; all non-variables had fixed interpretations. It has been common to trace the key notion of model theory, *satisfiability-in-a-structure*, or *truth-in-a-model*, back to Tarski’s seminal paper (Tarski, 1935) on the concept of truth in formalized languages. Wilfrid Hodges (1985/6) argues that it was only in the 1950’s that Tarski introduced interpretation relative to models. But Hodges’ arguments are disputed by Niiniluoto (1994) and by Feferman (2004), who finds the notion of *truth-in-a-structure* present implicitly in Tarski’s work as early as 1931.

In any case, model theory revolutionized semantics. This comes out most clearly (for linguists) when we look at the cascade of advances that came with the study of generalized quantifiers, a few of which will be described in Section 4. There had been work on generalized quantifiers by Mostowski (1957) and Lindstrom (1966) before Montague’s work, but for formal semanticists the source of generalized quantifiers was Montague and David Lewis⁹.

4. Generalized Quantifiers

**Montague’s work on quantifiers: background.** Montague, a student of Tarski’s, contributed greatly to the development of formal semantics with his development of intensional logic and his combination of pragmatics with intensional logic (Montague, present, that the compositionality of Tarski’s semantics for first-order logic was a model for Montague’s work on natural language.

⁹David Lewis presented the idea in a talk in 1969 and Montague in 1970; see (Lewis, 1970) and (Montague, 1973). They were colleagues, and no one seems to know whether they developed the idea independently, or together, or whether Montague got the idea from Lewis. Probably because Montague’s semantic program was more comprehensive, his work on this topic is cited more often.
His higher order typed intensional logic unified modal logic, tense logic, and the logic of the propositional attitudes, extending the work of Carnap (1956), Church (1951), and Kaplan (1964), putting together Frege’s function-argument structure with the treatment of intensions as functions to extensions.

Montague treated both worlds and times as components of "indices", and intensions as functions from indices (not just possible worlds) to extensions. The strategy of “add more indices” was taken from Dana Scott’s “Advice on modal logic” (Scott, 1970), an underground classic long before it was published.

The Fregean principle of compositionality was central to Montague’s theory and remains central in formal semantics. Montague showed that one could give a model-theoretic semantics for ordinary English, with a syntax rather close to surface structure; his higher-order typed logic\(^\text{10}\) was crucial for making that possible. The treatment of English noun phrases as uniformly denoting generalized quantifiers was one of the most vivid examples of that; that achievement made a big impression on linguists\(^\text{11}\).

**Montague and generalized quantifiers:** According to Peters and Westerståhl (2006), the logical notion of quantifiers as second-order relations is “discernible” in Aristotle, full-fledged in Frege, then forgotten until rediscovered by model theorists. Mostowski (1957) introduced unary generalized quantifiers, denoting sets of sets; these can capture the meanings of quantified expressions like *everything, something, an infinite number of things, most things*. It was Lindström (1966) who introduced binary generalized quantifiers, without which one can express *Most things walk*, but not *Most cats walk*. What we are now accustomed to calling ‘generalized quantifiers’, e.g. the denotation of the NP\(^\text{12}\) *most cats*, represents the application of a Lindström binary quantifier *most*, syntactically a Determiner, to its first argument *cats*, syntactically a Noun (or Common Noun Phrase), giving a unary generalized quantifier *most cats*.

Montague (1973) (and D. Lewis, 1970) proposed that English NPs like *every man, most cats* can be treated categorically, uniformly, and compositionally if they are interpreted as generalized quantifiers. This was a big part of the refutation of the point Russell and Strawson (and Chomsky) were agreed on, that there is no logic of natural language. That refutation opened a floodgate, and the next decade saw a great surge of work by linguists and philosophers, individually and together.

**Generalized quantifiers and English syntax.** Montague’s work showed how with a higher-typed logic and the lambda-calculus (or other ways to talk about functions), NPs could in principle be uniformly interpreted as generalized quantifiers (sets of sets). And Determiners could then be interpreted uniformly as functions that apply to common noun phrase meanings (sets) to make generalized quantifiers.

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\(^\text{10}\) Montague’s views of natural language type structure had a number of influences, including categorial grammar as developed by Polish logicians; he cited (Ajdukiewicz, 1960).

\(^\text{11}\) In my own case, it was one of the things that made me devote a good number of years to finding ways to integrate Montague’s work into linguistics. The enterprise started out as “Montague Grammar” (Partee, 1973, 1975, 1976), and developed into the more inclusive field of formal semantics.

\(^\text{12}\) In much western syntax, the Determiner is now considered the head of the Noun Phrase (Abney, 1987), which has been rechristened Determiner Phrase (DP). In this paper I continue to call it an NP.
Recall how we asked “Where’s the meaning of every man in (10), the first-order formalization of Every man walks?” Now, with Montague’s work, we have a semantic type, <<e,t>,t>, sets of sets of entities, to correspond to English NPs. In a simple sentence, the subject NP is the function, and the Verb Phrase (VP) is its <e,t>-type argument, as shown in (12). And although Montague treated the determiner every syncategorematically, that was inessential; every can be analyzed as in (12d).

(12)  

(a) Every student \( \lambda.P \forall x[\text{student}(x) \rightarrow P(x)] \) \( \text{type} <<e,t>,t> \)

(b) walks \( \text{walk} \) \( \text{type} <e,t> \)

(c) Every student walks \( \lambda.P \forall x[\text{student}(x) \rightarrow P(x)](\text{walk}) \) \( \text{type} t \)

≡ \( \forall x[\text{student}(x) \rightarrow \text{walk}(x)] \)

(d) every \( \lambda.Q \lambda.P \forall x[Q(x) \rightarrow P(x)] \) \( <<e,t>,<<e,t>,t>> \)

Montague’s interpretation of the sentence Every man walks is the same as Russell’s; the big difference is that Montague derives the interpretation compositionally; the semantic structure is homomorphic to the syntactic structure.

Martin Stokhof (2006), in describing the PTQ model of Montague grammar, isolates “two core principles that are responsible for its remarkable and lasting influence”:

A. Semantics is syntax-driven, syntax is semantically motivated.

B. Semantics is model-theoretic.

Montague did not invent model-theoretic semantics; but it was through his work that the model-theoretic approach became more widely known and adopted among linguists, with far-reaching changes to the field of linguistic semantics.

**Generalized quantifier theory and model theory.** Barwise and Cooper, a logician and a linguist, cooperated in the first major investigation of properties of determiners (Barwise & Cooper, 1981), studying English noun phrases from the perspective of the model-theoretic properties of generalized quantifiers and the determiners that help to build them.

A determiner like every denotes a function of type \( <<e,t>,<<e,t>,t>> \) as seen in (12d) above. The determiner’s first argument is the common noun: \([\text{every}][([\text{student}])].\) This generalized quantifier is itself of type \( <<e,t>,t> \), and takes the denotation of the VP as its argument: \([\text{every}][([\text{student}])][[\text{walks}]]\). Schematically the structure of a simple sentence of that sort can be represented as D(A)(B).

One of the first determiner universals discovered by Barwise and Cooper is the “Conservativity” universal expressed in (13):

(13) **Conservativity Universal:** Natural language determiners denote conservative functions.

(14) **Definition:** A determiner meaning D is conservative iff for all A,B, D(A)(B) = D(A)(A \( \cap \) B).

(15) **Examples:** No solution is perfect = No solution is a perfect solution. Exactly three cups are blue = Exactly three cups are blue cups. Every boy is singing = Every boy is a boy who is singing.
For example, no language has a (non-conservative) determiner $D$ such that $D$ members are excluded would mean All non-members are excluded. And if the word only in only boys were a determiner, it would be an example of a non-conservative determiner, since the false sentence Only males are astronauts is not equivalent to the true sentence Only males are male astronauts. But only is not a determiner; in general it combines with an expression of category X to make a new expression of category X, and in examples where it seems to combine with a noun to make an NP, it is really combining with an NP to make another NP, as it does in only that boy, only John, only John and the teacher.

The Conservativity Universal suggests a reason why it is useful for languages to have determiners that combine with a noun to make a generalized quantifier. The Conservativity Universal tells us that when evaluating $D(A)(B)$, we only need to consider A’s, never non-A’s. And the “A” position in that formula corresponds to the noun student in every student walks. So the noun indicates the domain of entities that are relevant to the truth of any sentence of the form $D$ NP VP. So not only are natural languages not “illogical”; the more we uncover about how the compositional semantics works, the more well-designed (well-evolved) natural languages turn out to be.

Barwise and Cooper had many other results of interest to linguists and logicians. They found a first good approximation to a formalization of the distinction between “weak” determiners, which can occur in there-sentences like (16a), and “strong” determiners, which cannot (16b).

(16)  
a. There are some/three/several/many/no unicorns in the garden.
b. *There are both/the/those/all unicorns in the garden.

(17)  
Key definitions:
a. A determiner $D$ is positive strong if $D(A)(A)$ is true whenever $D(A)$ is defined (A any subset of the universe).
b. $D$ is negative strong if $D(A)(A)$ is false whenever $D(A)$ is defined.
c. $D$ is weak if it is neither positive strong nor negative strong.

For a fuller discussion, and many more interesting properties of determiners and generalized quantifiers, see (Keenan, 2003; Keenan & Westerståhl, 1997; Larson, 1995; Westerståhl, 2011).

5. Quantifiers and pragmatics

Quantifiers also played an important role in the development of formal pragmatics and the recognition of the necessarily close connection between formal semantics and formal pragmatics.

One interesting domain where this can be seen is in the history of work on “Negative polarity items” or NPI’s. These are linguistic expressions like ever and at all and some uses of any, which sound fine in negative sentences but not in simple positive sentences.

\[\text{\footnotesize 13 I am not claiming that natural languages are the best medium for “doing logic”; I subscribe to Frege’s statement that a formal language provides an aid to natural language the way a microscope aids the eye.}\]

\[\text{\footnotesize 14 An asterisk in front of a sentence indicates that the sentence is ill-formed.}\]
(18)  a. John doesn’t ever eat any vegetables at all.
     b. *John ever eats any vegetables at all.

A “negative” determiner like no in subject NP licenses NPIs as in (19a), but a “positive”
determiner like some does not, as seen in (19b).

(19)  a. No boy had ever seen any problem at all in any of her proposals.
     b. *Some boy had ever seen any problem at all in any of her proposals.

For a long time most linguists had thought that what was crucial for allowing NPIs in a
sentence was the presence of some “negative” word or morpheme in a suitable structural
position – hence the name “Negative polarity items”. But it was known that there were
contexts without any overtly negative elements that nevertheless allowed NPIs:
comparative clauses, as in (20a); the antecedent but not the consequent of a conditional,
as in (20b,c), and in the first argument but not the second argument of the determiner
every as in (20d,e).

(20)  a. Mary answered more questions than anyone had ever answered before.
     b. If you ever have any problem at all, please let me know.
     c. *If John is lazy, he will have any problems at all.
     d. Every child who ever went there received some gifts.
     e. *Every child who went there at least once received any gifts.

That last pair, (20d,e), came as a surprise when it was discovered by Ladusaw (1979),
since previously linguists had thought that determiners could be divided into negative
ones like no that do allow NPIs and positive ones like some that do not. Ladusaw
identified the crucial model-theoretic generalization stated in (21).

(21)  **NPI Generalization (Ladusaw):** NPIs can occur in expressions that form part of
the argument of a monotone decreasing function.

Ladusaw’s generalization was the first example of a linguistic puzzle solved by model-
theoretic means that could not be mimicked by some more “syntactic” tree-like
representation at some level of “logical form”, the way one can syntactically show scope
relations and function-argument structure. The crucial property of being a monotone
decreasing function has no “logical form” representation; it is an inherently model-
theoretic property.

Ladusaw’s generalization was a major milestone in formal semantics, and at the same
time opened up two important problems for further work. (i) He gave no account of the
meanings of the NPIs themselves, treating them as having simple existential meanings.
The meaning of any in the examples in (20) is treated as basically the same as the
meaning of a or some, restricted to narrow scope under its “licensing” function
expression. (ii) The licensing of NPIs is sometimes sensitive to inferential properties in
context, and not only to properties of the semantic content of the sentence. Examples like
(22a-b), from (W. A. Ladusaw, 1996) were first raised by Linebarger (1987; 1980), to
argue that “weakly” negative expressions do not license NPIs directly but only in virtue
of negative implicatures that they contribute to the communicative context.
Pragmatics was eventually agreed to be a crucial part of the story not only in the solution of that second problem, ‘contextual licensing of NPIs’, but also in the analysis of the content of the polarity items themselves. Kadmon and Landman (1990) argued for a unified treatment of NPI any and free choice any (as in *Any doctor will recommend more exercise*), and for the need to supplement Ladusaw’s account with more about the meaning of any itself. They argued that the meaning of any is like that of a plus semantic/pragmatic conditions that reduce tolerance for exceptions: “widening”, and “strengthening”. Michael Israel (1996, 1998) has more recently extended that kind of account with further explicit pragmatics. I am omitting details, but I mention this to point out one important domain where the analysis of quantifiers (and in this case, not only quantifiers, although the determiners every, any, and no have played a leading role) has figured in a major turning point in the history of formal semantics, namely the recognition that formal semantics and formal pragmatics have to be studied together. One cannot just do semantics in isolation, and then take pragmatics to involve just the interaction of semantically interpreted sentences with the contexts in which they are used.

Quantifiers also starred in work that led to some major rethinking of the relation between semantics and pragmatics in the 1980’s and the move to dynamic semantics. (These developments are described more fully in several places, including (B. H. Partee with H. L. W. Hendriks, 1997).) The work of Kamp and Heim beginning in the early 1980’s was one of the major developments in the semantics of noun phrases, quantification, and anaphora. And more generally, their work influenced the shift from a “static” to a “dynamic” conception of meaning.

At the time of their work, indefinites like *a student* had been puzzling for a long time. Frege didn’t treat them. Russell analyzed sentences containing indefinites as quantified sentences with an existential quantifier. Early work in formal semantics by Montague, Barwise and Cooper, and others absorbed that view of indefinites into Generalized Quantifier theory, analyzing *a student* as in (23).

(23) a. *A student* $\lambda P \exists x [\text{student}(x) \& P(x)]$ type $<\text{e,t,t}>$

But indefinites do not behave like other quantifier phrases.

(i) Singular indefinites permit discourse anaphora, unlike *every boy, no boy*.

(24) a. *A boy came in. He was whistling.*
   b. *No/every boy came in. He was whistling.*

(ii) Singular indefinites have ‘variable quantificational force’ and figure in puzzling anaphoric relations in Geach’s famous donkey sentences (Geach, 1962).

(25) a. *Every farmer who owns a donkey beats it.*
   b. *If a farmer owns a donkey, he always beats it.*
Kamp (1981) and Heim (1982, 1983) offered solutions to these classic problems; on their theories, indefinite noun phrases are interpreted as variables (in the relevant argument position) plus open sentences, rather than as quantifier phrases. The puzzle about why an indefinite NP seems to be interpreted as existential in simple sentences but universal in the antecedents of conditionals stops being localized on the noun phrase itself. Its apparently varying interpretations are explained in terms of the larger properties of the structures in which it occurs, which contribute explicit or implicit unselective binders that bind everything they find free within their scope.

Both Kamp and Heim make a major distinction between quantificational and non-quantificational NPs; the semantics of indefinites, definites, pronouns and names is on their analysis fundamentally different from the semantics of “genuinely quantificational” NPs like every student. The diversification of NP semantics, and the unification of some kinds of determiner quantification with some kinds of adverbial quantification in examples like (25a-b), represented an important challenge to Montague’s uniform assignment of semantic types to syntactic categories, and in particular to the uniform treatment of NPs as generalized quantifiers. That challenge motivated work on type shifting and type-driven interpretation (Klein & Sag, 1985; B. H. Partee, 1986); and Kamp’s challenges to compositionality in his Discourse Representation Theory led first to the competing development of “Dynamic Montague Grammar” (Groenendijk & Stokhof, 1990, 1991) and then to a resolution (Muskens, 1993).

6. Quantifiers, universals, and typology
There is much more to say about quantifiers in the history of formal semantics. Much of the early history that I have described is very directly concerned with the relation of logic to language, and to the interplay between new ideas in linguistic theory and new ideas and new formal tools in logic that together have helped to bridge the seeming disparities between “logical form” and “linguistic form”. I will close with a few remarks about later developments more internal to linguistics, as formal semantics has become sufficiently “naturalized” within the field to be applied to such traditional linguistic concerns as language typology and the question of how much in language is universal and in what ways languages differ from one another.

The study of universals and typology is much farther advanced in phonology, morphology, and syntax than in semantics, but formal semantics has made advances in that domain in recent decades, and there have probably been more advances in the study of quantification than in any other area.

Bach, Jelinek, Kratzer, and Partee (eds.) (1995) was the first major work on typology from the perspective of formal semantics. One of the questions that motivated our work was whether all natural languages have NPs that are interpreted as generalized quantifiers. Barwise and Cooper (1981) had hypothesized “Yes”; we marshaled our colleagues to help us answer the question, and it turned out to be “No”. At least as widespread, but maybe also not universal, is “adverbial quantification” as in (26) and (25b), first studied by David Lewis (1975), and important in the work of Kamp and Heim.

(26) A quadratic equation usually has two distinct roots.
some of the earliest work on semantic universals was Barwise and Cooper’s work on Determiner universals, discussed above. Since that time, there has been much more work on universals and typology of determiners and quantifiers, including several recent important works (Keenan & Paperno, 2012; Peters & Westerståhl, 2006; Szabolcsi, 2010).

I have left many exciting recent developments unmentioned; as history moves closer to the present, research multiplies and one can hardly keep up with it, much less discuss it in a short article. It was not surprising that quantification was one of the first topics to be explored by linguists, logicians and philosophers working together. What may be more surprising is that even as formal semantics expands its reach into many more aspects of language, research on quantification is as active and innovative as ever.

References


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